Lesson Plan (Linda Bolin)

Lesson Title: Ratios, Unit Rates, and Proportions
Course: Pre-algebra
Date: December Lesson 1

Utah State Core Content and Process Standards:
2.1 Model and illustrate meaning of ratios and proportions including percent
2.1a Compare ratios to determine if they are equivalent
2.1b Compare ratios using the unit rate

Lesson Objective(s): Compare quantities using ratios, rates, and proportions

Enduring Understanding (Big Ideas):
Rates and Ratios are comparisons using division

Essential Questions:
- How is a ratio or rate used to compare two quantities or values? Where can examples of ratios and rates be found?
- How can I model and represent rates, ratios, and proportions?
- What is a proportion?
- How are cross products and unit rates helpful in determining whether two ratios are equivalent?

Skill Focus:
Write ratios, rates and proportions. Compare quantities using these.

Vocabulary Focus:
Ratio, unit rate, words that suggest rate: per, each, and at (@), numerator, denominator, proportion

Materials:
- Clock with seconds hand or timer, 1 piece of bubble gum for each team, 2 2-liter bottles of soda per class, 1 tennis or racquet ball for each team, 1 deck of cards per student pair, 2 meter sticks and tape for each team
- Worksheets: Rate Contests, Comparing Numbers, Equivalent Ratios-Proportions
- Linker Cubes
- Journal pages: Defining Rate, Defining Proportion

Assessment (Traditional/Authentic): Performance tasks, journal

Ways to Gain/Maintain Attention (Primacy):
Manipulative, conjecture, calculators, games and contests, writing, cooperative activity literature

Written Assignment:
- Journal pages: Defining Rate, Defining Proportion
- Worksheets: Rate Contests, Comparing Numbers, Equivalent Ratios-Proportions
- McDougal Littell p.272 #22-29, 37 or other practice

Post vocabulary on the board

Content Chunks

Starter:

1. Find the decimal for each
   a) $\frac{3}{5}$  
   b) $\frac{1}{8}$  
   c) $\frac{10}{4}$

2. Write each percent a fraction and a decimal.
   a) 40%  
   b) 120%  
   c) 2.5%
Launch-Mobilizing Background Knowledge:

In our language, we use words to show a comparison such as larger, faster, and darker. Write three other words that indicate a comparison.

In math, we can use two operations to show a comparison. We can use subtraction, or finding the difference as in \(12 - 3 = 9\) (twelve is nine greater than three), or we can use division, or finding a quotient, as in \(12 \div 3 = 4\) (twelve is four times greater than three). For each of the following tell whether the comparison is being made with subtraction or with division:

A. \(8 - 2 = 6\)
B. 10 is twice as great as 5
C. 4 is three less than 7
D. \(6/2 = 3\)

Lesson Segment 1: How can a ratio be used to compare two quantities or values? Where can examples of ratios be found?

Smart Pal communicators: Have student pairs work together to show both ways to compare the following pairs of numbers (subtraction, finding a difference and, division, finding a quotient), showing their reasoning on a Smart Pal or team board. Have them write the comparisons for each as shown:

\[
10 - 4 = 6 \\
10/4 = 2.5
\]

And then write words for their comparison as shown:

Ten is six more than four
10 is 2.5 times greater than 4.

A. 6 and 3  B. 20 and 5  C. 18 and 2  D. 15 and 10

Work together as a class using Linking Cubes and the “Comparing Numbers” worksheet to compare quantities. Tell students they will be using ratios for making comparisons in this unit.

Lesson Segment 3: What is a rate? Where can examples of rates be found?

A rate is a special kind of ratio. When we need to use a rate, we use words like: per, each, or at (@). Explain that rates are used when the two items being compared in a ratio are different measures or units. For example, comparing the number of green apples to red apples, boys to girls, or trucks to cars on the road does not require us to use different measures just different numbers. However, comparing the number of CD’s to the cost of CD’s is comparing number measures to money measures. Comparing miles to gallons also require different unit measures. Have students use the Agree, Disagree or Unsure response cards attached to tell whether or not each of the following situations involves two different units of measure. As they hold up their responses, remind them that different units of measure are written using a special ratio called a rate.
A. The number of miles you travel in an hour
B. The number of white pairs of shoes you own compared to the number of black.
C. The length of a rectangle compared to the width
D. The amount of money you make in an 8 hour day.

When we are comparing different measures or units, we use words like **per** and symbols like /. Show the examples m/g or price per CD, points per quarter, m/h, and beats per measure (music). We find unit rates by setting up a ratio and dividing it.

Do **Math Talk** to practice the following. In Math Talk, a student and partner go to the board. They work on the board as the others in the room are working at their desks. When they finish, **teacher** selects one of them to explain what they wrote as the class listens. As they listen to the explanation, the class members each write one good question they could ask. A good question is a question that begins with “why” or “how”. One or two of the class member are called on to ask their good question to the partners at the board. This is an excellent way to get kids listening to each others explanations.

Practice these example:
A. 125 miles on 8 gallons of gas. Write as a ratio and find the rate of miles **per** gallon.
B. 90 points in a basketball game. This is an average of how many points in each quarter? Write ratio and find the rate of points **in each** quarter.
C. 5 CD’s for $60. This is buying CD’s **at** how many dollars each?
D. Your heart beats 80 times in a minute. How many beats per second is this?

**Practice Activities: Discuss and complete the following activities together**

**Rate Contests**

Help students do the rate activities described on the Rate Contests worksheet. Use words like per, each and at (@) in the discussion that ensues.

**Journal:** Work with students to complete the “Defining Rate” (Frayer Model) vocabulary page for their journals.

**Lesson segment 3: What is a proportion? How are cross products and unit rates helpful in determining whether two ratios are equivalent?**

Give student pairs one card (attached). Any left over cards should be place where students can have access to look at them. Have students mix around the room looking for another pair whose ratio is equivalent to theirs. If they don’t find other students, they should check the extra cards that were laid out. When they find the card they are looking for, they discuss with the other pair how they determine equivalency. All four will need to be able to explain how they decided their ratios were equivalent. Give them five minutes to find their match. Have a couple of students explain how they knew their ratios were equivalent. Model and have students use calculators to divide several equivalent ratios to determine if they are a proportion. Also, model checking for proportionality using cross products.
Find student pairs from the matching activity that had an inverse. (3/4 and 4/3, 2/5 and 5/2, 4/5 and 5/4, 1/6 and 6/1). Q. These have the same numbers, so are they equivalent ratios? How can you use equivalent quotients or cross products to check?

Work through the Equivalent Ratios- Proportions worksheet using the Reporting Reporter cooperative structure. Student teams work together to answer all the parts of problem 1. Teacher selects a student from each team to be the reporter. This person travels to another team to describe how their team answered the question. The neighboring team listens and adds any ideas they may have had, or makes any suggestions or corrections needed. The reporter then returns to their home team and the teams repeat this with the remaining problems on the worksheet.

Journal: Have students complete the “Defining Proportion” Frayer Model for their journal.

Help them read their proportions “____ compares to ____ the same way that ____ compares to ____”. Or, ____ divides into ____ the same number of times that ____ divides into ____. They should write the words for their proportion examples in the column 3 of the journal.

Assign text practice for finding unit rates or determining equivalent ratios as needed.
Sketch Linker Cube trains to compare the numbers first by **subtracting**, then by **dividing**. When we compare two numbers by dividing, we say they are a **ratio**. Write the **ratio** three ways (:, to, /).

1. **30, 10**
   a) Find the **difference**.   
      ____________________________  
   b) Find the **quotient**.   
      ____________________________  

Write the **ratio** three ways:

2. **20, 4**
   a) Find the **difference**.   
      ____________________________  
   b) Find the **quotient**.   
      ____________________________  

Write the **ratio** three ways:

3. **12, 9**
   a) Find the **difference**.   
      ____________________________  
   b) Find the **quotient**.   
      ____________________________  

Write the **ratio** three ways:

4. **15, 12**
   a) Find the **difference**.   
      ____________________________  
   b) Find the **quotient**.   
      ____________________________  

Write the **ratio** three ways:

5. In your own words explain what a ratio is.
**Rate Contests**

Name____________________  
Date____

**A. Bubble Gum Blowing:** Assign a counter, a timer, a blower and a scribe. Blow as many bubbles as possible in two minutes.

1. Write a ratio comparing the number of bubbles and the number of seconds in two minutes.

2. Find your blower’s rate of bubbles per second. Then, find the rates for the other teams in the class.

3. Explain how you know which team won by using their rate per second rather than their total number of bubbles.

4. About how many bubbles would you expect to have blown if the contest had continued for 10 minutes. Explain how you decided that number.

**B. Ball Bouncing contest:** Assign a timer, a bouncer, a counter and a scribe. Bounce a ball and catch it waist high for 20 seconds. Rotate roles so each team member has a turn to bounce and catch for 20 seconds.

5. Write the ratio of bounces caught in 20 seconds for each person on your team. Find each person’s rate per second.

6. If you had continued bouncing and catching for a minute at the same rate, how many bounces caught would you have? Write your rate per minute. Explain how you came up with that rate per minute.

**Soda Guzzling:** Two class members will guzzle. Two other class members will pour to keep a few three ounce Dixie cups about 2/3 full on a table. Guzzlers will guzzle for 30 seconds.

7. Write a ratio for each guzzler using ounces to 30 seconds. Find each guzzler’s rate per seconds.
8. If a 2 liter soda bottle contains approximately 67 ounces, how long would it take each guzzler to drink the entire bottle at their rate? Show all your reasoning.

**High Jumper:** Measure your height in centimeters. Tape a couple of meter sticks to a wall beginning at 100 centimeters up, so they extend upward well above reach. You will need a recorder and 2 sighters. Each person begins at a stand still and jumps as high as possible touching the meter stick. Jumpers may crouch, but may not run for momentum. The sighters use a pointer or another meter stick to mark the touch spot and say how many centimeters high that was, and the recorder writes the ratio of height jumped to height of jumper.

9. Find the rate for centimeters jumped per inch of your height.

10. Write the ratio and find the rate for the others in your group. Compare rates to determine who can jump the highest for their height.

**Heart Rate:** Find pulse in your neck or wrist. Time keeper times for six seconds as students count heart beats. Multiply the number of beats by 10 to get beats per minute (60 seconds). Next do jumping jacks or running in place for 1 minute, stop, find pulse. Repeat timing and counting.

11. What was your resting heart rate? Active heart rate?

12. Write the rate for someone in the class whose resting rate was lower than yours, and someone whose resting rate was higher than yours. Do this for active rate too.

Lower resting _____ higher resting _____ lower active _____ higher active _____

**A Better Buy-Shopping War:** Two players: Using a card deck, each player picks two cards. The first is the price for an item in dollars. The second is the number of items purchased. Each player finds the rate for each item. The player with the better buy, keeps all four cards. Ties remain on the table until the next hand is played. Player with the most cards at the end of the deck, wins. (Aces are 1, Kings are 0, Queens are 12, Jacks are 11).

13. Not counting zeros, describe which pairs of cards in the deck would give you the better buy. Explain why this rate is a better buy.
1. Write five word sentences that suggest a rate using words like per, each, at (@).

2. Write five rates using the math symbol /
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Some ratios have the same quotient as other ratios. When two ratios have the same quotient, they are called a proportion.

1. Find the quotient for each ratio in the pair. Tell if the two ratios are a proportion.
   A. \( \frac{5}{10}, \frac{6}{12} \)
   B. \( \frac{8}{10}, \frac{4}{5} \)
   C. \( \frac{3}{4}, \frac{5}{6} \)

2. Find the cross products for each ratio in the pair. Tell if the two ratios are a proportion.
   A. \( \frac{7}{21}, \frac{8}{12} \)
   B. \( \frac{4}{32}, \frac{5}{40} \)
   C. \( \frac{8}{10}, \frac{12}{15} \)

3. Find a number that produces the same quotient or equal cross products, so you will have a proportion.
   A. \( \frac{4}{6} = \frac{12}{?} \)
   B. \( \frac{?}{4} = \frac{15}{6} \)
   C. \( \frac{12}{20} = \frac{?}{5} \)

4. List three ratios that are in proportion to \( \frac{3}{5} \)
1. Write three examples of proportions using the numbers: 1, 4, 2, and 8.

2. Rearranging the numbers in numerators and denominators, so you have two different proportions. Making certain each comparison is still a proportion.

   a. \( \frac{3}{5} = \frac{6}{10} \)  
   b. \( \frac{2}{6} = \frac{1}{3} \)  
   c. \( \frac{5}{10} = \frac{3}{6} \)
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